

ONE MAN, 3,312 VOTES:  
A MATHEMATICAL ANALYSIS OF  
THE ELECTORAL COLLEGE

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*The significant standard for measuring . . . voting power, as Mr. Banzhaf points out, is . . . his [voting member's] "ability \*\*\*, by his vote, to affect the passage or defeat of a measure". . . .*

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*In order to measure the mathematical voting power . . . it would be necessary to have the opinions of experts based upon computer analyses.<sup>1</sup>*

I. INTRODUCTION

**I**N THE WAKE of the Supreme Court's reapportionment decisions Congress is now seriously considering several proposals to abolish the Electoral College and to replace it with one of a number of alternative plans.<sup>2</sup> The ensuing discussion of this issue has been heated, and even the United States Chamber of Commerce and the American Bar Association have taken public stands.<sup>3</sup> Sentiment for some change in

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1. *Iannucci v. Board of Supervisors*, 20 N.Y.2d 244, 251-53, 229 N.E.2d 195, 198-99, 282 N.Y.S.2d 502, 507-09 (1967).

2. For the actual bills and an explanation of the proposed amendments see pp. 317-22 *infra*.

3. One interesting application of the reapportionment decisions in this area was a suit by the State of Delaware in the United States Supreme Court against all of the other states. *Delaware v. New York*, 385 U.S. 895 (1966). Although Delaware recognized that the entire Electoral College system could be modified only by constitutional amendment, it asked the Court to outlaw the unit-vote or "winner take all" system of casting electoral votes by which all of a state's votes, by law, go to the candidate receiving the greatest number of popular votes. It claimed that the effect of that system was to disadvantage voters in the less populous states. The arguments were based in part on the author's techniques. See Motion for Leave to File Complaint, Complaint and Brief at 83-85, *Delaware v. New York*, 385 U.S. 895 (1966). The Supreme Court declined to hear the case, apparently on procedural grounds.

This analysis supports Delaware's claim that its citizens have far less than average voting power but for reasons other than those stated in the complaint and brief. However it also shows that two remedies tentatively proposed — a proportional or a district system — would only create new and even greater inequalities.

3. The Chamber of Commerce of the United States, following a policy referendum of its member organizations, Jan. 31 [1966] announced it favored abolishing the electoral college and shifting to either a nationwide popular vote or a district system of choosing Presidential electors. The final vote of the Chamber members for approving the new policy position was 3,877 (91.5 percent) in favor and 362 (8.5 percent) opposed.

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A special Commission on Electoral College Reform of the American Bar Association recommended in January 1967 that the President and Vice President be elected as a team by a national popular vote and that the Electoral College be abolished. The Commission terms the present system "archaic, undemocratic, complex,

the present system is also running strong among politicians and the man-in-the-street.<sup>4</sup> Interestingly enough, however, the workings and effects of the existing system are so poorly understood that many people are actually taking positions on the issue which appear to be contrary to their own interests.<sup>5</sup> Others appear to have recognized the effects of the present system but lack both proof and a quantitative appreciation of the magnitude of the problems.<sup>6</sup>

The critical point of departure for an evaluation of the current method of presidential election or of any of the alternatives now under consideration is a determination of the voting power of the individual voter in the different states. This article has adopted, as a measure of voting power, a technique<sup>7</sup> which has been generally accepted by mathematicians and political scientists and which is also beginning to gain judicial approval.<sup>8</sup> In order to apply this measurement technique to

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ambiguous, indirect, and dangerous" and said that a direct election would ensure election of the popular winner, abolish the "anachronism and threat" of the runaway elector, "minimize the effect of accident and fraud in controlling the outcome of an entire election" and "put a premium on voter turnout and encourage increased political activity throughout the country." Commission on Electoral College Reform, American Bar Ass'n, *Electing the President*, 53 A.B.A.J. 219, 220 (1967). See also COMMITTEE ON FEDERAL LEGISLATION, BAR ASS'N OF THE CITY OF NEW YORK, PROPOSED CONSTITUTIONAL AMENDMENT PROVIDING FOR DIRECT ELECTION OF PRESIDENT AND VICE PRESIDENT (1967).

4. The Gallup Poll May 18 reported a heavy majority of the American people would like to do away with the electoral college and substitute a direct vote of the people for President. A nationwide sampling of citizens was asked, "Would you approve or disapprove of an amendment to the Constitution which would do away with the electoral college and base the election of a President on the total popular vote cast throughout the nation?" Results: approve, 63 percent; disapprove, 20 percent; no opinion, 17 percent.

5. Sen. Quentin N. Burdick (D N.D.) conducted a poll of all members of the 50 state legislatures and found 58.8 percent in favor of direct election of the President and only 9.7 percent in favor of continuing the existing system. Another 21.2 percent favored a proportional method of casting state electoral votes, while 10.2 percent were in favor of a district system. Results of the Burdick poll showed that in 44 states, direct election was favored by 50 percent or more of the legislators responding. . . . Support for direct election was almost as strong among legislators from small states as those from large states. . . . Burdick's conclusion was that "the climate for the reform of the electoral college has never been more favorable."

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6. For example the National Grange, which represents principally the rural interests from the less populous states, took a position in favor of the present system and against the direct election because it felt that the former favored its members. Statement of Harry L. Graham, in *Hearings Before the Senate Subcomm. on Constitutional Amendments*, 90th Cong., 1st Sess. (July 12, 1967). See also note 37 *infra*.

7. These include the American Bar Association, the Association of the Bar of the City of New York, and Alexander M. Bickel, Chancellor Kent Professor of Law and Legal History at Yale Law School. See pp. 326-27 *infra*.

8. For other applications see Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 YALE L.J. 1309 (1966); Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 RUTGERS L. REV. 317 (1965).

9. See generally pp. 309-12 *infra*. Prior to testifying as an expert witness in a reapportionment case involving both weighted voting and multi-member districts, the author talked with four of the leading scholars in the field of the mathematical analysis of voting power. These are Samuel Krislov, Professor of Political Science at the University of Minnesota, who analyzed weighted voting in New York

the question of the voting power of citizens in each state, the aid of the Fairleigh Dickinson University computer system was enlisted.<sup>9</sup> The title of the present article reflects the results of the computer analysis in which it was determined that a voter in New York State has 3.312 times the voting power of a citizen in another part of the country. The complete analysis clearly demonstrates that the current Electoral College system falls short of even an approximation of equality in voting power. Such a disparity in favor of the citizens of New York and the other large states also repudiates the often voiced view that the inequalities in the present system favor the residents of the less populous states. Moreover, these existing inequalities in voting power appear to be reflected in the allocation of campaign resources, the selection of presidential candidates, and in the effectiveness of minority groups and third parties.

It can also be shown by utilization of the same measurement technique that there would be even greater inequalities in voting power if either of the two present proposals for modifying the indirect system of election were adopted; *i.e.*, if the President were to be elected under either a proportional or district plan. The third proposal of direct election presents a generically different approach which obviates the inequalities inherent in an indirect election. This article will explain the method for the measurement of voting power, analyze, with the aid of a computer, the voting power of citizens in the different states under the existing and proposed plans for electing the President, and examine some of the ramifications and implications of this analysis.

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City's Board of Estimate, see notes 15 & 19 *infra*; Irwin Mann of the Courant Institute of Mathematics at New York University and the coauthor of three previous mathematical analyses of the electoral college, see note 14 *infra*; William H. Riker, Chairman of the Department of Political Science at the University of Rochester, who has analyzed voting power in several different situations, see notes 14, 15, 16, & 20 *infra*; and Lloyd S. Shapley, Senior Research Mathematician at the RAND Corp., who is coauthor of the Shapley-Shubik index of voting power and who worked with Mann on three previous analyses of the Electoral College, see notes 14, 15, & 20 *infra*. These scholars were kind enough to submit affidavits in support of the author's technique of analysis and testimony, each of which said in part:

There is a method for the measurement of voting power in various voting situations which is recognized and generally accepted in the fields of mathematics and political science. This method is reflected in articles by the authors cited above and more specifically in the articles cited in footnotes 14-21 of Banzhaf's article in the Yale Law Journal. . . .

Given the mathematical model described above, I believe that the analysis of voting power reported in Banzhaf's article in Yale Law Journal is, within the limitations and conditions stated therein, a reasonable, consistent, and correct analysis of the distribution of voting power among the citizen voters, and that it is both consistent with and a reasonable extension of the generally recognized techniques for the measurement of voting power.

Affidavits submitted in *Town of Greenburgh v. Board of Supervisors*, 49 Misc. 2d 116, 266 N.Y.S.2d 998 (Sup. Ct. 1966).

9. The computer calculations upon which this article is based were performed under the direction of the author by Professor Martin A. Jacobs, Department of Mathematics, Fairleigh Dickinson University, Teaneck, N.J., on the University's G.E. 235 Time-Shared Computer System. However, see note 28 *infra*.

## II. THE MEASUREMENT OF VOTING POWER

This technique for the measurement of voting power is based on the simple and almost self-evident proposition that the purpose of any voting system is to allow each voting member some chance, however small, to affect the decisions which must be made. It can be demonstrated by resort to common experience that different methods of voting can result in variations in the effectiveness of the vote of a given individual. In some cases, for example the election of a club president in which each member may cast one vote, each member has the same chance to affect the outcome, and all members obviously have equal voting power. In other situations, for example in a stockholders' meeting where members' cast votes in proportion to their stock holdings, the ability of the voting members to affect the outcome is not equal and they do not have equal voting power.

Voting power is simply the ability to affect decisions through the process of voting. It can be most easily measured by comparing the opportunities each voter has to affect the outcome. If all voters have an equal chance to affect the outcome in a given voting situation, we say that they have equal voting power. However, if some voters have a greater chance than others, we say that the voting powers are unequal.

In any voting situation it is possible to consider all of the possible ways in which the different voters could vote; *i.e.*, to imagine all possible voting combinations. One then asks in how many of these voting combinations can each voter affect the outcome by changing his vote. Since, *a priori*, all voting combinations are equally likely and therefore equally significant, the number of combinations in which each voter can change the outcome by changing his vote serves as the measure of his voting power. In other words, no one can say which voting combinations will occur most often, or which combinations will predominate as to the more important issues. The most a legislature or judge can do in seeking to equalize voting power is to satisfy himself that the system allows each voting member an opportunity to affect the outcome in an equal number of equally likely voting combinations.

A person's voting power, then, is measured by the fraction of the total number of possible voting combinations in which he can, by changing his vote, alter the outcome of the group's decision. To be more precise, the ratio of the voting power of voter *X* to the voting power of voter *Y* is equal to the ratio of the number of possible voting combinations in which *X* could alter the outcome by changing his vote (assuming that no other voters change their votes) to the number of possible voting combinations in which *Y* could alter the outcome by

changing his vote (also assuming no other voters change their votes). This method of measuring voting power, which is described here only in briefest form, has been discussed at greater length by the author in prior studies which dealt primarily with the reapportionment decisions.<sup>10</sup>

It is important to recognize that this technique measures the voting power of the individual which is inherent in the rules governing the voting system and the distribution of population, and does not reflect the actual ability that any given individual voter has in a particular election to affect the outcome. The latter would depend to some extent on factors which are not inherent in the system, such as the relative power of the political parties in different geographical areas, and conditions which may be peculiar to the voter himself (*e.g.*, whether as a sign of protest he decides to vote for a minority party candidate who has no chance of winning). Thus, a critical distinction must be drawn between inequalities in voting power which are built into the system (*e.g.*, the old county unit system in Georgia<sup>11</sup> or the distribution of electoral votes in the Electoral College) and those which result either from the free choice among citizens as to how they use their voting power (*e.g.*, the political impotence of a Republican in a solidly Democratic state) or from factors outside of the legal rules governing the process (*e.g.*, voter intimidation, weather, the televised prediction of election results, etc.). Concededly, these and other external factors may affect a citizen's ability to affect the outcome of an election, and, therefore, the theoretical voting power of an individual may not coincide with his actual ability to affect the outcome of any particular election. The voting power measured here is that inherent in the system and necessarily represents an average of a voter's effectiveness in a large number of equally likely voting situations. However, it is only with respect to those inequalities which result from the rules of a particular system of voting on which we may properly focus attention in determining the basic "fairness" of the system itself.<sup>12</sup>

10. See note 7 *supra*.

11. See *Gray v. Sanders*, 372 U.S. 368 (1963).

12. The inequalities which this article discloses actually result from two factors: the electoral system and the unequal distribution of population among the states. But the 1960 census figures are used for the purpose of illustration only and inequalities will continue to exist as long as there are large and small states. In contrast to population inequalities, such factors as the solid South, liberal and minority power in the larger states, and rural domination of the House of Representatives are probably too transitory to form the basis for a lasting "balance of power" solution to the problem of electing the President.

This technique may also be used as an analytic tool to evaluate the effect of other political factors on voting. See Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 *YALE L.J.* 1309, 1333-34 (1966).

### III. ACCEPTANCE OF THE MEASURING TECHNIQUE

No doubt some critics will object to this method for measuring voting power, either because it is "too theoretical," or because it is based on an examination of only a limited number of situations which occur infrequently, or for some other reason. Considerations of space do not permit anything like a complete justification.<sup>13</sup> Moreover, once it is agreed that voting power is a relevant criteria, a reasonable quantitative definition of the term is required and objections to any one measuring technique may be weighed only as such a technique may prove to be more or less acceptable than some other reasonably precise and workable definition. Nevertheless, some very persuasive evidence for the validity of the technique described is the acceptance of the basic principle by other disciplines and the recent judicial approval which it has received.

The technique of measuring voting power by calculating the number of opportunities each voter has to affect the decision has been generally accepted in the fields of mathematics<sup>14</sup> and political science.<sup>15</sup>

13. The briefest answer to the "too theoretical" argument is that voting power is itself a theoretical quantity which can only be demonstrated by theoretical techniques. See p. 323 *infra*. The results of any given election also reflect the workings of such other factors as the candidates themselves and the issues and may even depend on such things as the weather (which often affects voter turnout). Moreover the effects which have been demonstrated have been suspected by a large number of serious thinkers for some time and are in accord with empirical evidence. See pp. 323-28 *infra*.

It is true that this measure of voting power is based on a small number of voting combinations — those where the balance can be shifted easily — but this is because these are the ones which are critical and decisive. Many properties are measured by selecting either the most decisive and critical point (the strength of a rope is measured at its breaking point) or a number of selections which are considered average or a reasonable sample; e.g., intelligence or aptitude tests, the Dow Jones stock index, and the Cost of Living Index.

14. See, e.g., J. KEMENY, J. SNELL & G. THOMPSON, INTRODUCTION TO FINITE MATHEMATICS 74, 108 (1957); Shapley, *Solution of Compound Games*, in *ADVANCES IN GAMES THEORY* 267 (1964); Riker, *A Test of the Adequacy of the Power Index*, 4 *BEHAVIORAL SCI.* 120-31 (1959); Shapley, *Simple Games: An Outline of the Descriptive Theory*, 7 *BEHAVIORAL SCI.* 59 (1962); Shapley, *A Value for N-Person Games*, 2 *ANNALS OF MATHEMATICAL STUDIES* 307 (1953).

For reports of mathematical applications of this technique to a different aspect of the Electoral College see Mann & Shapley, *The A Priori Voting Strength of the Electoral College*, in *GAME THEORY AND RELATED APPROACHES TO SOCIAL BEHAVIOR* 151 (M. Shubik ed. 1964); I. MANN & L. SHAPLEY, *VALUES OF LARGE GAMES VI: EVALUATING THE ELECTORAL COLLEGE EXACTLY* (RAND Corp. Memo RM-3158-PR, May 1962); I. MANN & L. SHAPLEY, *VALUE OF LARGE GAMES IV: EVALUATING THE ELECTORAL COLLEGE BY MONTE CARLO TECHNIQUES* (RAND Corp. Memo. RM-2651, Sept. 19, 1960). *But see* note 48 *infra*.

15. See, e.g., P. DAVID, R. GOLDMAN & R. BAIN, *THE POLITICS OF NATIONAL PARTY CONVENTIONS* 174-75 (1960); G. SCHUBERT, *JUDICIAL BEHAVIOR* (1964); G. SCHUBERT, *QUANTITATIVE ANALYSIS OF JUDICIAL BEHAVIOR*, ch. 4 (1959); W. Riker, *Bargaining in a Three-Person Game*, Sept. 6-10, 1966 (paper delivered at the 1966 Annual Meeting of the American Political Science Association); Krislov, *Theoretical Attempts at Predicting Judicial Behavior*, 79 *HARV. L. REV.* 1573 (1966); Krislov, *Power and Coalition in a Nine-Man Body*, 6 *AM. BEHAVIORAL SCI.* Apr. 1963, at 24; Riker, *Some Ambiguities in the Notion of Power*, 58 *AM. POL. SCI. REV.* 341 (1964);

Moreover, the technique has been specifically employed to analyze voting power in Congress,<sup>16</sup> stockholders' meetings,<sup>17</sup> the French Assembly,<sup>18</sup> New York City's Board of Estimate,<sup>19</sup> weighted voting situations in general,<sup>20</sup> including the New Jersey Senate<sup>21</sup> and Nassau County, New York,<sup>22</sup> and in multi-member districts.<sup>23</sup> In at least two cases courts have ordered computer analyses of weighted voting plans before them and have held the plans unconstitutional because of hidden inequalities uncovered by the studies.<sup>24</sup> Very recently, in *Iannucci v. Board of Supervisors*,<sup>25</sup> New York State's highest court, adopting some of the author's arguments as articulated in an amicus curiae brief, ruled against two weighted voting plans, and held that such plans must be subjected to a mathematical-computer analysis. The court stated:

Although the small towns in a county would be separately represented on the board, each might actually be less able to affect the passage of legislation than if the county were divided into districts of equal population with equal representation on the board and several of the smaller towns were joined together in a single district. (See Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 Rutgers L. Rev. 317.) The significant standard for measuring a legislator's voting power, as Mr. Banzhaf points out, is not the number or fraction of votes which he may cast but, rather, his "ability \* \* \*, by his vote, to affect the passage or defeat of a measure." . . . And he goes on to demonstrate that a weighted voting plan, while apparently distributing this voting power in proportion to population, may actually operate to deprive the smaller towns of what little voting power they possess, to such an extent that some of them might be completely disen-

Schubert, *The Power of Organized Minorities in a Small Group*, 9 ADVANCED SCI. Q. 133 (1964); Shapley & Shubik, *A Method for Evaluating the Distribution of Power in a Committee System*, 48 AM. POL. SCI. REV. 787 (1954).

16. Riker & Niemi, *The Stability of Coalitions on Roll Calls in the House of Representatives*, 56 AM. POL. SCI. REV. 58 (1962); Shapley & Shubik, *supra* note 15, at 787-90.

17. Shapley & Shubik, *supra* note 15, at 791.

18. Riker, *supra* note 14, at 122-31.

19. Krislov, *The Power Index, Reapportionment and the Principle of One Man, One Vote*, 1965 MODERN USES OF LAW & LOGIC 37, 40-43.

20. Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 YALE L.J. 1309 (1966); Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 RUTGERS L. REV. 317 (1965); Krislov, *supra* note 19; W. RIKER & L. SHAPLEY, *WEIGHTED VOTING: A MATHEMATICAL ANALYSIS FOR INSTRUMENTAL JUDGMENTS* (RAND Corp. Memo. P-3318, 1966).

21. Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 RUTGERS L. REV. 317, 335-38 (1965).

22. *Id.* at 338-40.

23. Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 YALE L.J. 1309 (1966).

24. *Dobish v. Board of Supervisors*, 53 Misc.2d 732, 279 N.Y.S.2d 565 (Sup. Ct. 1967); *Morris v. Board of Supervisors*, 50 Misc.2d 929, 273 N.Y.S.2d 454 (Sup. Ct. 1966).

25. 20 N.Y.2d 244, 229 N.E.2d 195, 282 N.Y.S.2d 502 (1967).

franchised and rendered incapable of affecting any legislative determinations at all. . . .

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Unfortunately, it is not readily apparent on its face whether either of the plans before us meets the constitutional standard. Nor will practical experience in the use of such plans furnish relevant data since the sole criterion is the mathematical voting power which each legislator possesses in theory — i.e., the indicia of representation — and not the actual voting power he possesses in fact — i.e., the indicia of influence. In order to measure the mathematical voting power of each member of these county boards of supervisors and compare it with the proportion of the population which he represents, it would be necessary to have the opinions of experts based on computer analyses. The plans, then, are of doubtful constitutional validity and to establish the facts one way or another would be, in all likelihood, most expensive. In our view, it was incumbent upon the boards to come forward with the requisite proof that the plans were not defective.

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. . . [A] considered judgment [of the plans] is impossible without computer analyses and, accordingly . . . there is no alternative but to require them [the boards] to come forward with such analyses and demonstrate the validity of their reapportionment plans.<sup>26</sup>

This decision is of particular interest for several reasons. First, it is particularly persuasive because it comes from one of the most highly regarded state courts in the country. It is all the more persuasive because of the experience that this court has gained in this specialized area because for a variety of reasons New York courts have dealt with a large number of weighted voting cases. Secondly, the decision is particularly important because it expressly recognizes the validity of mathematical analysis in this area and specifically holds that it may be the determining factor regardless of other empirical evidence. Finally the decision is of greatest interest here because it validates this technique of measurement by equating it with the legal meaning of

26. *Id.* at 251, 252-53, 254, 229 N.E.2d at 198, 199, 200, 282 N.Y.S.2d at 507, 508-09, 510. For other courts citing the author's work see, e.g., *Kilgarlin v. Hill*, 386 U.S. 120, 125 (1966); *WMCA, Inc. v. Lomenzo*, 246 F. Supp. 953, 959-60 (S.D.N.Y. 1965) (Levet, J. dissenting); *Graham v. Board of Supervisors*, 18 N.Y.2d 672, 219 N.E.2d 870, 273 N.Y.S.2d 419 (1966); *Dobish v. Board of Supervisors*, 53 Misc.2d 732, 279 N.Y.S.2d 565 (Sup. Ct. 1967); *Davis v. Board of Supervisors*, 51 Misc.2d 347, 273 N.Y.S.2d 133 (Sup. Ct.), *rev'd*, 28 App. Div. 583, 279 N.Y.S.2d 434 (1967); *Town of Greenburgh v. Board of Supervisors*, 49 Misc.2d 116, 266 N.Y.S.2d 998 (Sup. Ct. 1966); *Barzelay & Carocci v. Board of Supervisors*, 47 Misc.2d 1013, 263 N.Y.S.2d 854 (Sup. Ct. 1965).



voting power. It is not unlikely that other courts will now follow the lead and likewise adopt this same technique — a technique which has been noted with interest by the United States Supreme Court.<sup>27</sup>

IV. ANALYSIS

The following study analyzes, in accordance with the measurement principles which have already been discussed, the voting power of citizens under the current electoral system and under the three proposed plans for the election of the President now being considered by Congress. The general method utilized in each case is to determine the ability of a citizen in one state to affect the outcome of an election and to compare this with the like ability of a citizen of another state in the same election. The calculations actually employ census figures rather than the number of voters because electoral votes are allocated on that basis.

A. *The Existing Electoral College*

The analysis of the existing Electoral College is a two step process. First, all of the different possible arrangements of *electoral* votes are examined with the aid of a computer, and a determination is made of those arrangements in which any given state, by a change in the way it casts its bloc of electoral votes, could change the outcome of the election.<sup>28</sup> As the second step one looks to the people of the state, and determines in how many of the different voting combinations involving people of that state any given resident could affect, by changing his vote, how that state's bloc of electoral votes would be cast. Finally,

27. *Kilgarlin v. Hill*, 386 U.S. 120, 125 (1966).

28. The calculations for the first step of the analysis of the present Electoral College were actually done in two slightly different ways, using two essentially similar techniques. The first was performed by Lloyd S. Shapley of the Rand Corp. and Irwin Mann, who is now connected with the Courant Institute of Mathematics in New York City (both of whom also assisted the author in formulating the material in this paper). They used an IBM 7094 computer and a specially developed program which used the so-called Shapley-Shubik index of voting power, I. MANN & L. SHAPLEY, *VALUES OF LARGE GAMES VI: EVALUATING THE ELECTORAL COLLEGE EXACTLY* (RAND Corp. Memo. RM-3158, 1962). Similar calculations were also made by the IBM New York Scientific Center with the cooperation of Mr. Robert G. Loomis and Mr. Lee Papayanopoulos on an IBM System/360 Model 50 computer using a program specially developed to use the measurement technique suggested by the author in Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 *RUTGERS L. REV.* 317 (1965). See R. Sharashian, *An Algorithm for Obtaining the Frequency Distribution of a Linear Function of n 0-1 Variables; An Application to Weighted Voting* (unpublished).

The differences between the two techniques is explained in Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 *YALE L.J.* 1309, 1318n.21 (1966). However, as predicted by W. RIKER & L. SHAPLEY, *WEIGHTED VOTING: A MATHEMATICAL ANALYSIS FOR INSTRUMENTAL JUDGMENTS* 9n.2 (RAND Corp. P-3318, 1966), and by Mann, in personal conversation with the author, the two techniques yield substantially similar results for large numbers of voting units. Indeed, the differences between these two sets of calculations were at most a few percent. Table I of the Appendix is based on the figures from Mann and Shapley.

the results of the two steps are combined and the result represents the chance that any voter has of affecting the election of the President through the medium of his state's electoral votes; in other words, his chance to effectively participate in the presidential election. Although the chance that any given voter will cast a decisive vote in the presidential election is extremely small, the relative ability of each voter to cast such a vote serves as a meaningful measure — and perhaps the only possible measure — of his potential voting power.

The results of an analysis of the present Electoral College are shown in Table I of the Appendix. The first column contains the name of the state (or the District of Columbia), the second its population according to the 1960 census, and the third the number of its electoral votes. The fourth column shows the relative voting powers of the citizens of the fifty states and the District of Columbia. The figures have been normalized or adjusted so that the figure for citizens with the least voting power is set equal to one, and all others are numbers greater than one. The table shows, for example, that a citizen of New York has 3.312 times the voting power of a citizen of the District of Columbia. Looking at the same figure from another point of view, a resident of the District of Columbia has less than one-third the chance that a citizen of New York has of casting a decisive vote in the presidential election. The fifth column shows the extent of these inequalities; the percentage by which the votes of the citizens of other states are more effective than the votes of the most deprived voters. Finally, the sixth column shows the percentage deviation from the average, an average obtained by adding the effective voting powers in the fourth column and dividing by fifty-one.<sup>29</sup>

The general effect of the existing system is easily seen from the table. Citizens of the small and medium-sized states are severely deprived of voting power in comparison with the residents of the few very populous states who have far more voting power than the others. The present Electoral College system, in conjunction with state imposed unit-vote ("winner take all") laws, in effect in all of the states,<sup>30</sup> greatly favors the citizens of the most populous states and deprives citizens of the less populous states of an equal chance to affect the election of the President.

29. An equally "valid" average might be obtained by multiplying the voting power by the number of voters in each state and dividing by the total number of voters. This would be a per voter average rather than the per state average which has been used in these tables. See Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle*, 75 YALE L.J. 1309, 1330n.41 (1966).

30. See, e.g., CAL. ELECTIONS CODE §§ 10204-05 (West, 1961); N.Y. ELECTION LAW 290 (McKinney 1964); OHIO REV. CODE ANN. § 3505.10 (Page 1960); PA. STAT. ANN. tit. 25, § 3056(f) (1963); VA. CODE ANN. §§ 24-290.4-6 (1964).

The reason for the deprivation is not so easily perceived. It arises from the fact that the influence of a citizen-voter vis-à-vis his state's electoral votes does not vary as the simple inverse of (one divided by) the population of the state. In other words, the ability of a voter to affect the way his state's bloc of electoral votes will be cast is not halved if the population is doubled nor divided by three if the population is tripled. Rather, an examination of all of the possible voting combinations reveals that the chance of a voter to affect the way his state's electoral votes will be cast decreases as the inverse of (one divided by) the square root of the population. Thus, by forcing the citizens of each state to vote as a bloc, the system increases the voting power of the residents of the larger states.

This effect may be illustrated by a simplified example. Consider the election of a congressman from a single electoral district. Every voter has 1 vote and, therefore, has equal voting power in this election. Suppose, however, that 3 voters, *A*, *B*, and *C*, were for some reason required to vote as a group and that a bloc of 3 votes would be cast in accordance with the majority vote of *A*, *B*, and *C*. There are 8 different voting combinations in which *A*, *B*, and *C* may cast their votes.<sup>31</sup> In 4 of these *A* can alter the way in which the bloc of 3 votes will be cast by changing his own vote. The situation is the same for *B* and for *C*; each can change the outcome in half of the total number of combinations. Thus, each of the three can, by changing his own vote, affect the way in which the bloc of 3 votes will be cast in 50% of the cases.<sup>32</sup>

31. *A* may vote in either of 2 ways. In each of these 2 cases *B* could also vote in either of 2 ways. Finally, in each of these 4 cases, *C* could vote in either of 2 ways for a total of 8. In general, a body of *N* persons able to vote for either of two candidates has  $2^N$  possible voting combinations. This example reflects the usual case where the voters have a choice between two major candidates.

32. The table below shows the 8 possible voting combinations. It also shows the combinations in which each voter can cast a critical vote. It should be noted that often more than one voter can cast a critical vote in the same combination in the sense that a change by either would affect the outcome.

	Possible Voting Combinations			Winning Candidate	Voter Casts a Critical Vote		
	<i>A</i>	<i>B</i>	<i>C</i>		<i>A</i>	<i>B</i>	<i>C</i>
1	Y	Y	Y	Y			
2	Y	Y	X	Y	C	C	
3	Y	X	Y	Y	C		C
4	Y	X	X	X		C	C
5	X	Y	Y	Y		C	C
6	X	Y	X	X	C		C
7	X	X	Y	X	C	C	
8	X	X	X	X			

Y = a vote for candidate Y  
 X = a vote for candidate X

Y = Y wins  
 X = X wins

C means that a particular voter can change the outcome in that combination

In contrast, any single elector in the district has 100% control over how his smaller "bloc" of 1 vote should be cast. Since the bloc of 3 votes is three times as effective from the point of view of affecting the election as any single vote, and since *A* can affect the way in which the bloc of 3 votes will be cast in 50% of the cases, *A* has more voting power than other voters who have only 100% control over 1 vote; *i.e.*, 50% of 3 votes is greater than 100% of 1 vote. The voting power of *A*, *B*, and *C* has been increased by requiring them to cast their votes as a bloc. Bloc or unit-voting can, therefore, be identified as the crucial factor resulting in the disparities in voting power under the present system.

This example may be extended to further demonstrate the effect. Suppose that 5 voters in the same electoral district, *P*, *Q*, *R*, *S*, and *T*, were required to vote as a group and their group vote determined how the bloc of 5 votes would be cast. There are 32 possible voting combinations, and *P* can affect the outcome in 12 of these. *P* thus has a  $\frac{12}{32}$  chance of affecting 5 votes which gives him more than 1.8 times the voting power of the ordinary voter who has 100% control over only 1 vote; *i.e.*,  $\frac{12}{32}$  of 5 is 1.8, which is greater than 100% of 1. By extending this process it can be shown that the voting power of an individual voter increases as the size of his voting group increases<sup>33</sup> and

33. In a group of *N* voters, there are  $2^N$  different voting combinations. For an individual voter to be able to cast a critical vote, the other voters in the group must be equally divided. The formula for the number of combinations which can be formed by *M* persons divided into two equal groups is:

$$\frac{M!}{\left(\frac{M}{2}\right)! \cdot \left(\frac{M}{2}\right)!}$$

The exclamation point (!) indicates a factorial. It means that the number it follows is to be multiplied by every positive integer smaller than itself (*e.g.*,  $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ). In calculating the number of times each person can cast a critical vote, the fraction must be multiplied by 2 to account for votes for the two different candidates. Calculations for four different sized voting groups are presented below.

No. in Group and No. of Votes	Possible Combinations	Number of Times Each Casts Critical Vote	Percent Influence	Effective Voting Power
3	$2^3 = 8$	$2 \times \frac{2!}{(1!) \cdot (1!)} = 4$	$4/8 = 50\%$	50% of 3 = 1.5
5	$2^5 = 32$	$2 \times \frac{4!}{(2!) \cdot (2!)} = 12$	$12/32 = 37.5\%$	37.5% of 5 = 1.88
7	$2^7 = 128$	$2 \times \frac{6!}{(3!) \cdot (3!)} = 40$	$40/128 = 31.2\%$	31.2% of 7 = 2.18
9	$2^9 = 512$	$2 \times \frac{8!}{(4!) \cdot (4!)} = 140$	$140/512 = 27.3\%$	27.3% of 9 = 2.46

that for numbers of reasonable size the voting power is proportional to the square root of the group's population (where the group controls a number of votes proportional to its number of members). There may also be voting situations in which different numbers of people are required to vote as a unit but where each voting unit may not have a number of votes proportional to its population. In such a case, the ability of any member of the group to affect the group's decision (or how it will cast its votes) will nevertheless decrease in the fashion which has been previously indicated, which for numbers of reasonable size is as the inverse of the square root of the population.<sup>34</sup> Thus, by combining these two principles, it can be shown that the voting power of any member of a reasonably sized voting group will be proportional to the voting power of the group (which is roughly proportional to the number of votes it can cast) and inversely proportional to the square root of its population.

34. Consider a group made up of  $N + 1$  citizen-voters, where  $N$  is an even number. The total number of ways in which each citizen-voter could vote for either of two principal candidates is  $2^{N+1}$  which is equal to  $2 \cdot 2^N$ . Each would be able to cast a critical vote only where the other  $N$  citizen-voters were equally divided into two groups;  $\frac{N}{2}$  voting yea and  $\frac{N}{2}$  voting nay. This, as previously indicated, can happen

in  $\frac{2 \cdot N!}{\left(\left[\frac{N}{2}\right]!\right) \cdot \left(\left[\frac{N}{2}\right]!\right)}$  ways. Thus, an individual citizen-voter would be critical

in determining the outcome of a vote in the following fraction of combinations.

$$\frac{2 \cdot N!}{\left(\left[\frac{N}{2}\right]!\right) \cdot \left(\left[\frac{N}{2}\right]!\right)} \div (2 \cdot 2^N) = \frac{N!}{2^N \cdot \left(\left[\frac{N}{2}\right]!\right) \cdot \left(\left[\frac{N}{2}\right]!\right)}$$

The factorial of large numbers may be very closely approximated by the following formula which is known as Stirling's formula:

$$m! = e^{-m} \cdot m^m \cdot \sqrt{2\pi m}$$

where  $e$  and  $\pi$  are well known mathematical constants. Substituting this value into the previous formula by allowing  $\frac{N}{2}$  to equal  $m$ , the fraction becomes:

$$\frac{e^{-2m} \cdot (2m)^{2m} \cdot \sqrt{4\pi m}}{2^{2m} \cdot e^{-m} \cdot m^m \cdot \sqrt{2\pi m} \cdot e^{-m} \cdot m^m \cdot \sqrt{2\pi m}}$$

By expanding the numerator into separate multiplicative terms and cancelling common terms in the numerator and denominator, the result is that the individual citizen-voter

is critical in  $\frac{1}{\sqrt{2\pi N}}$  combinations.

Where  $m$  is greater than 100, Stirling's formula gives a result which is accurate to within 0.1% and becomes more accurate as  $m$  increases. I. SOROLNIKOFF & R. REDHEFFER, *MATHEMATICS OF PHYSICS AND MODERN ENGINEERING* 644-45 (1958).

Riker and Shapley have reported that voting power also varies as the inverse of the square root of population under the Shapley-Shubik index. W. RIKER & L. SHAPLEY,

As an example of the operation of these principles in their application to the Electoral College, consider the states of New York and Alaska. New York has approximately seventy-four times the population of Alaska. One might suppose that a citizen of New York would have one-seventy-fourth the chance of affecting New York's 43 electoral votes as a voter in Alaska would have of affecting Alaska's 3 votes. However, as has been shown, the relative effectiveness of the two voters depends instead on the ratio of the square roots of the populations, and, therefore, a New Yorker has about one-ninth as much chance of affecting his state's electoral votes as a voter in Alaska has of affecting his. But, because a New Yorker may potentially affect 43 votes as compared with the Alaskan's potential effect on only 3 votes, the New Yorker's decrease in effectiveness with respect to his state's electoral votes is far outweighed by the vastly larger number of electoral votes he may potentially affect. Actually, a New Yorker has almost twice the potential for affecting the overall election as does an Alaskan voting on the same day in the same election (3.312 compared with 1.838).

This comparative analysis has been performed identically for all of the other states. The most favored citizens under the present Electoral College are those of New York, California, Pennsylvania, and Ohio; the most deprived are those in Maine, New Mexico, Nebraska, Utah, and the District of Columbia. Citizens in 32 states (and the District of Columbia), only two fewer than are needed to propose a constitutional amendment, have less than average voting power.<sup>35</sup> Citizens of all other states are at a disadvantage in comparison with the voting power of citizens of the most favored state, New York.

It should be noted that the plan and suggested amendment proposed by President Johnson<sup>36</sup> would in no way correct the inequalities which have been demonstrated. Under the plan, the actual electors would be abolished but the states would retain the electoral votes they now have. All of the electoral votes of each state would be cast for the candidate receiving the greatest number of votes in that state. Since this plan constitutes only a formal change, leaving the substance of the

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WEIGHTED VOTING: A MATHEMATICAL ANALYSIS FOR INSTRUMENTAL JUDGMENTS (RAND Corp. Memo. P-3318, 1966).

35. The author has often been asked, by people who believe that the average always falls somewhere in the middle, how more than half the states can be below average. The answer is that it will always happen when the distribution of figures is not uniform (i.e., is "uneven") and the higher figures are, in non-mathematical language, "far out of line." Consider a school class of 20 children each 5 years old taught by two 27 year old teachers. The average age of all of the people in the room is 7:  $20 \times 5 + 2 \times 27 = 154$ ;  $154/22 = 7$ . In this example all of the 20 children are below the average age. The example would not be substantially changed if the children's ages ranged from 4 to 7 years.

36. See President Johnson's message to Congress on January 20, 1966, reported in 1966 CONG. Q. 1248. Now S.J. Res. 21, 90th Cong., 1st Sess. (1967) (introduced by Senator McGee of Wyoming).

current indirect method unaltered, the voters in the larger states would still be able to exert an inordinate amount of influence by affecting their state's large bloc of electoral votes, and the result would be that the voters in the small and middle-sized states would still have far less than equal voting power.

President Johnson's proposal, however, would have one important effect. It would freeze the present inequalities and put them beyond the reach of the legislatures of the several states and the federal courts. Under the present system, the inequalities exist because of the unit-vote ("winner take all") laws which are in force in the states. These laws are still susceptible to change by act of the legislatures — possibly with the encouragement of some form of interstate compact — or by the courts. If the inequalities were locked in by constitutional amendment, subsequent correction would require the joint efforts of Congress and the legislatures of three-fourths of the states.

### B. *Proportional Plans*

Under the so-called proportional plans<sup>37</sup> for election of the President, the states would retain their present number of electoral votes. However, instead of awarding all of the votes in a state to the candidate receiving the greatest statewide vote — as is now the case in all fifty states — the state's electoral vote would be divided in proportion to the statewide popular vote for the candidates. Thus if a state had 3 electoral votes and 60% of the people voted for candidate X, candidate X would receive 60% of 3 votes or 1.8 electoral votes.

The effects of such plans may be more easily seen by considering the mathematical equivalent of this system. Significantly, under this method, every time a citizen casts his vote he directly causes a shift of a small portion of the state's electoral vote. This small fraction is equal to the total number of the state's electoral votes divided by the total number of voters in the state. Thus, in a hypothetical state with 1,000,000 voters and 4 electoral votes, each citizen's vote in the election would, in effect, shift  $4/1,000,000$  or .000004 electoral votes in the modified Electoral College. Thus, each citizen-voter would actually cast a vote in a pseudo-electoral college, a vote which constitutes a small fraction of a single electoral vote.

37. See, e.g., S.J. Res. 84, 90th Cong., 1st Sess. (1967) (introduced by Senator Sparkman of Alabama and others); S.J. Res. 7, 90th Cong., 1st Sess. (1967) (introduced by Senator Holland of Florida); S.J. Res. 3, 90th Cong., 1st Sess. (1967) (introduced by Senator Smathers of Florida).

It is interesting to note that if this type of plan is adopted the citizens of these states will suffer significant deprivations of voting power both with respect to a direct election and with respect to the present system.

There would be no inequalities under a proportional system if the vote of each citizen-voter in the United States shifted the same fractional vote in the pseudo-electoral college. In other words, all citizens would have equal voting power if the fractional votes effectively cast by each citizen in the pseudo-electoral college were equal. Such would be the case, for example, if the electoral votes were distributed in direct proportion to the population of the states. In fact, however, this is far from true.

Every state is now entitled to two electors as well as an additional number of electors based on the state's population. Moreover, each state is entitled to at least one elector in addition to the original two whether or not its population is equal to that of an average congressional district. Thus, no matter how small the population, each state has at least three electoral votes. It is also well known that smaller states have far more electoral votes per resident than the more populous states.<sup>38</sup>

Under the proportional system, voters in each state would not be forced to vote by groups and, as a result, groups would not have the power to shift large blocs of votes by majority control. Thus, the advantage the citizens of the larger states enjoy under the present system would disappear, and the greater number of electoral votes per unit of population would give an advantage to the citizens in the less populous states. Each voter, in effect, would cast a fractional vote in the Electoral College, and this vote would be equal to the number of the state's electoral votes divided by the state's population. Simple division demonstrates that the residents of the smaller states would have a far greater chance to affect the election of the President through their greater fractional votes than residents of the large and middle-sized states because, under these circumstances, voting power is directly proportional to the fractional vote each voter effectively casts. Table II of the Appendix indicates this result on a state by state basis.

Under the proportional system the most favored citizens would be those in Alaska whose votes would be worth more than five times as much as votes cast in California or New York. Moreover, 171,049,740 citizens in 36 states and the District of Columbia would have less than average voting power under the proportional plan. This is over 95% of the total population!

38. For this reason it was widely believed that the existing Electoral College favored the citizens of the smaller states. But this is not the case. Although the smaller states have more electoral votes per resident, this does not give them a net advantage under the existing system because the unit-vote rule requires voters in each state to vote as a group. As previously explained, where voters vote as groups, the large groups, whose majorities control large blocs of electoral votes, have a significant voting advantage which far outweighs the advantage which the smaller states thought they had obtained from the unequal distribution of electoral votes.



### C. District Plans

Under the proposed district plans for the election of the President, the citizens of each state would elect two electors at large and one from each congressional district, or at least the electoral votes would be cast as though electors had actually been so chosen.<sup>39</sup> Thus, although residents would still vote in units — electoral districts — in which the majority vote of the group would determine how that unit's vote would be cast, the voting units for the most part would be smaller and more nearly uniform than under the present system. Moreover, in contrast to the present electoral system, no voting unit would be capable of controlling a large bloc of electoral votes. For these reasons, residents of the larger states would not receive the voting power advantage they enjoy under the existing system.

A twofold similarity exists between the district plan and the proportional plan: each state would be entitled to at least three electors or electoral votes regardless of its population, and the smaller states would still have a far greater number of electors or electoral votes per unit of population than the more populous states. Under the proportional system, as previously indicated, the voting power of each citizen is directly proportional to the fractional electoral vote that he, in effect, controls; *i.e.*, proportional to the number of electoral votes of his state divided by the state's population. Here the practical effect would be almost the same although the mechanism is neither so direct nor so simple.

To calculate a citizen's voting power under the district system, one first calculates the chance that each citizen-voter would have to affect the election through his effect on the two at-large electoral votes. This figure, as previously demonstrated, is inversely proportional to the square root of the state's population. To this figure one then adds the chance that the voter would have to affect the election through his effect upon the electoral vote corresponding to the congressional district in which he resides. This is inversely proportional to the square root of the district's population. The resultant sum represents the chance each voter would have of affecting the outcome of the presidential election — his voting power.

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39. See, *e.g.*, S.J. Res. 86, 90th Cong., 1st Sess. (1967) (introduced by Senator Gruening of Alaska) (providing for electors bound to previously announced candidates); S.J. Res. 55, 90th Cong., 1st Sess. (1967) (introduced by Senator Cotton of New Hampshire) (providing for bound electors); S.J. Res. 25, 90th Cong., 1st Sess. (1967) (introduced by Senator Scott of Pennsylvania) (providing for electoral votes to be cast by the states in accordance with the greatest number of votes in each voting unit); S.J. Res. 12, 90th Cong., 1st Sess. (1967) (introduced by Senator Mundt of South Dakota and others) (providing for bound electors).

The results of the analysis of the district plan are shown in Table III of the Appendix. Similar to the proportional system, the voter in the small state would have a disproportionate advantage in voting power because the smaller states would have more electoral votes per unit of population. In making these calculations the simplifying assumption was made that within each state the population of each electoral district was equal to the state's population divided by the number of districts. Even with this assumption, which tends to underestimate the disparities, the inequalities in voting power are very significant. The voter in the small state is again favored at the expense of voters in the large and middle-sized states. Under the district plan the most favored voters would be those in Alaska, Nevada, and Wyoming, who would have more than two and one-half times the voting power of citizens in New York and California. Moreover, 168,014,360 citizens in 34 states would have less than average voting power under the district system. This is over 93% of the total population!<sup>40</sup>

If past and present conditions are any guide, it is not realistic to assume that districts will be even of approximately equal size. Variations of 20% and more are not uncommon and in the past variations have been far more considerable.<sup>41</sup> To the extent that there are variations in district population within a given state, the inequalities shown in the table would be increased. Although its effects are beyond the scope of this analysis, it is fair to point out that the district system also offers both the opportunity and motive for gerrymandering with respect to the presidential election — something which is not true of any other system.

#### D. Direct Presidential Election

There are also several proposals for election of the President by a direct vote of the people.<sup>42</sup> The winner would be the candidate for whom the greatest number of people voted, either initially or in a run-off election. Since no distinction whatever is made between votes cast

40. See note 35 *supra*.

41. In early 1964 the United States Supreme Court, holding that congressional districts within a state must be of substantially equal population, struck down the apportionment plan in Georgia which included variations of three to one. *Wesberry v. Sanders*, 376 U.S. 1 (1964). The variations in congressional districts in other states at that time were shown in an Appendix to the Opinion of Mr. Justice Harlan, dissenting. 376 U.S. at 49. More than three years after that decision, it is now estimated that congressional districts in 33 states still have variations of more than 10%. See U.S. CONG. CONG. & AD. NEWS, July 20, 1967, at iv.

42. See, e.g., S.J. Res. 15, 90th Cong., 1st Sess. (1967) (introduced by Senator Burdick of North Dakota); S.J. Res. 6, 90th Cong., 1st Sess. (1967) (introduced by Senator Smith of Maine and others); S.J. Res. 2, 90th Cong., 1st Sess. (1967) (introduced by Senator Bayh of Indiana on behalf of himself and 19 other senators).

by residents of different states or congressional districts, it is obvious that all voters would have an equal chance of affecting the outcome of the election and, therefore, would have equal voting power. For ease of comparison with the other plans, these equalities are shown in Table IV of the Appendix. Of all systems, both present and proposed, the direct election is the only plan which guarantees to each citizen the chance to participate equally in the election of the President.

Although the pragmatic effect of the direct election in equalizing voting power is clear, it is also well to note that several structural problems are also avoided by this method. First, because no citizen is arbitrarily required to vote as a member of a group (state or district), there are no inequalities in voting power caused by bloc-voting or unit-voting nor is there any possibility of inequalities caused by gerrymandering. Secondly, because electoral votes are abolished, there are no inequalities introduced because of an inequitable distribution of electoral votes, nor could any be introduced by a reassignment of electoral votes caused by population changes. Thirdly, the so-called phenomena of "wasted votes" is eliminated because each vote is counted directly in determining the results of the presidential election. It has been argued that under the Electoral College the votes of citizens voting in the minority of each state were "wasted" because they had no effect upon the final election but instead were cast aside after the first of two stages in which each state decided how its electoral votes would be cast.<sup>43</sup> Finally, the direct election would eliminate any possibility of a "misfire" in which a candidate might be elected after receiving less than a majority of the popular vote.<sup>44</sup>

43. The State of Delaware, in asking the Supreme Court to strike down the use of the unit-vote rule in the Electoral College, put it this way:

Votes cast for the losing candidate within a particular state are not only discarded at an intermediate stage of the elective process but are effectively treated as if they had been cast for an opponent. . . . The state unit system causes [minority] votes to be spent and their own political effectiveness exhausted at a preliminary stage of the election.

Motion for Leave to File Complaint, Complaint and Brief at 7-8, 56, *Delaware v. New York*, 385 U.S. 895 (1966).

See also note 2 *supra*.

44. The electoral college method of electing a President has governed forty-five presidential elections and has produced fourteen Presidents who did not obtain a majority of the popular votes cast in the election. They are: John Quincy Adams in 1824 (with 30.54 percent of the popular vote); James K. Polk in 1844 (49.56 percent); Zachary Taylor in 1848 (47.35 percent); James C. Buchanan in 1856 (45.63 percent); Abraham Lincoln in 1860 (39.79 percent); Rutherford B. Hayes in 1876 (48.04 percent); James A. Garfield in 1880 (48.32 percent); Grover Cleveland in 1884 (48.53 percent); Benjamin Harrison in 1888 (47.86 percent); Grover Cleveland in 1892 (46.04 percent); Woodrow Wilson in 1912 (41.85 percent); Woodrow Wilson in 1916 (49.26 percent); Harry S. Truman in 1948 (49.51 percent); and John F. Kennedy in 1960 (49.71 percent).

Of the fourteen minority Presidents, three of them each received fewer popular votes than his major opponent.

In 1824: Andrew Jackson received 43.13 percent of the popular vote and 37.93 percent of the electoral vote; John Q. Adams, 30.54 percent of the popular

## V. EFFECTS OF VOTING POWER INEQUALITIES

It has been shown that under the present Electoral College, as well as with the proportional and district plans, voters residing in different states have an unequal ability to affect the election of the President. Although this is something that can be demonstrated with mathematical precision, like other forms of political or social power it cannot be conclusively confirmed by empirical evidence. The New York Court of Appeals put it this way: "Nor will practical experience in the use of such plans furnish relevant data since the sole criterion is the mathematical voting power which each legislator possesses in theory — i.e., the indicia of representation — and not the actual voting power he possesses in fact — i.e., the indicia of influence."<sup>45</sup> Although one cannot empirically measure voting power, one may be able to observe some of the effects of its inequitable distribution, and thereby obtain some confirmation that the mathematical quantity he had determined with such precision is closely related to social and political phenomena which the processes of law can appreciate and with which they can deal.

Under the present system a New Yorker's vote has more than twice the chance of affecting the election of the President as the vote of a resident of Nebraska. Both mathematical analysis and common sense would indicate that an informed candidate would be willing to expend more than twice the resources per voter on a New Yorker as he would on a Nebraskan. In particular, one would expect higher per

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and 32.18 percent of the electoral; Henry Clay, 13.24 percent of the popular and 14.18 percent of the electoral; and William H. Crawford, 13.09 percent of the popular and 15.71 percent of the electoral. Because no candidate received a majority of the electoral vote, the selection of President devolved on the House of Representatives. Adams was selected President, receiving the votes of thirteen states to seven states for Jackson and four for Crawford. It is alleged that Adams won the election because of Clay's support.

*In 1876:* Democratic presidential candidate Samuel J. Tilden won a majority of the popular vote (50.99 percent). He had over 250,000 popular votes more than the Republican candidate, Rutherford B. Hayes. Yet Tilden lost the election by one electoral vote (185 to 184), after certain disputed electoral votes from four states had been determined (two days before Inauguration Day) adversely to him by an Electoral Commission created by Congress.

*In 1888:* Grover Cleveland received 48.66 percent of the popular vote and 42 percent of the electoral vote. On the other hand, Benjamin Harrison obtained a popular vote of 47.86 percent and an electoral majority of 58 percent. Harrison was elected President. Although Cleveland had about 100,000 popular votes more than Harrison, Harrison won pivotal states by small margins. A switch of a few thousand votes in New York would have swung the election to Cleveland.

COMMISSION ON ELECTORAL COLLEGE REFORM, AMERICAN BAR ASS'N, ELECTING THE PRESIDENT 27-28 (1967) (footnotes omitted).

Charles W. Bischoff of the Department of Economics, Massachusetts Institute of Technology, has calculated that the chances of a misfire remain great even in modern times and that in an election as close as that between Kennedy and Nixon, there was almost a 50% chance of the winner receiving less than half the popular vote. See N. PEIRCE, THE PEOPLE'S PRESIDENT — THE ELECTORAL COLLEGE IN AMERICAN HISTORY AND THE DIRECT VOTE ALTERNATIVE (to be published in spring, 1968).

45. *Iannucci v. Board of Supervisors*, 20 N.Y.2d 244, 252, 229 N.E.2d 195, 199, 282 N.Y.S.2d 502, 508 (1967).

capita expenditures of campaign finances as well as an inordinate amount of the presidential candidate's time and other resources (*e.g.*, attention by chief assistants and promises of potential patronage) to be allocated to the larger states. Since this in fact appears to be the case,<sup>46</sup> it provides a partial confirmation of the analysis. One would also expect, for similar reasons, that this would be one important factor in the decision of major parties to select candidates from the large states, an effect which is well documented and which has long been recognized.<sup>47</sup>

When third parties within states support one of the major party candidates, they have power and importance to the extent that in a reasonably close election their votes may be able to shift the outcome one way or another. Since individual members of third parties in the largest states have more than twice the voting power of members of third parties in more than half of the other states, both mathematical analysis and common sense also indicate that a third party in the larger states has more political power per member than third parties in most of the other states. To this extent, the inequitable distribution of voting power under the Electoral College may enhance the power and importance — and perhaps even the development — of third parties in the larger states.

## VI. SUMMARY AND CONCLUSIONS

1. It is possible to measure voting power by using a technique which is generally accepted in the fields of mathematics and political science and which is beginning to receive judicial approval. Under this method a citizen's voting power is proportional to his ability to affect the election by his vote.

2. The existing Electoral College system discriminates against voters in the small and middle-sized states by giving citizens of the

46. See p. 306 *supra*.

47. For example, in the last 25 presidential elections a New Yorker has been a candidate for 26 out of a possible 100 positions; an average of 1 New Yorker running in every election. Of the 46 men elected to the Presidency, 11 have been New Yorkers. COMMISSION ON ELECTORAL COLLEGE REFORM, AMERICAN BAR ASS'N, ELECTING THE PRESIDENT, appx. G & H (1967).

It is important to distinguish with respect to both the expenditure of resources and the selection of candidates between gross and per capita effects. Even if there were no voting inequalities, all other things being equal a candidate would probably tend to spend more campaign money in large states than in small states simply because there are more voters in each large state whom he may be able to persuade. However, if a New Yorker's vote has twice the chance of determining the election of the candidate as a Nebraskan's, a shrewd candidate will tend to spend more on each New York voter than on each Nebraskan. The ratio of resources expended in New York to resources expended in Nebraska will then tend to depend on the fact that New York has more voters and on the fact that they are each more valuable in terms of winning the election. Naturally the actual figures will also depend greatly on a large number of external factors which may vary from election to election. A similar double-barreled effect probably accounts in part for the very large number of New York candidates.

large states an excessive amount of voting power. Citizens of states like New York and California have over two and one-half times as much chance to affect the election of the President as residents of some of the smaller states and over three times as much chance as citizens of the District of Columbia. Disparities in voting power of over 200% have been demonstrated and disparities of over 100% are not uncommon. Citizens of 32 states and the District of Columbia have less than average voting power.

3. Proportional plans under which the electoral votes of each state would be divided in proportion to the candidates' state-wide vote discriminate against residents of the large and middle-sized states by giving citizens in the small states a greatly excessive voting power. Under such proposed plans, citizens of states like Nevada and Alaska would have more than four times as much chance to affect the election of the President as residents of New York or California. A disparity of over 400% has been demonstrated. Moreover, under such plans, 171,049,740 citizens of 36 states and the District of Columbia — over 95% of the total population — would have less than average voting power.

4. District plans, under which the citizens of each state would elect two electors at large and one from each congressional district, also discriminate against the large and middle-sized states by placing excessive voting power in the hands of citizens of the small states. Even under an ideal system in which congressional districts within each state would be equal in population, citizens of less populous states would have over two and one-half times the voting power of residents of some of the larger states. Disparities of over 100% are common. 168,104,360 citizens of 34 states — again over 90% of the total population — would have less than average voting power. Moreover, because the election would be based upon congressional districts which may not be uniform in population, citizens of the same state may not have the same voting power and, as a result, the disparities already demonstrated would be increased. Gerrymandering, which would be possible under this plan, is beyond the scope of this analysis.

5. In a direct presidential election all citizens would have equal voting power and an equal chance to affect the outcome of the election. No other existing or proposed plan can even approximate such equality.

6. As many people have long suspected, the inequalities in voting power under the present Electoral College may be reflected in such

practical matters as the allocation of campaign finances and other resources, the selection of candidates, and the effectiveness of third parties.

The analysis presented herein reveals, apparently for the first time,<sup>48</sup> the type and magnitude of the inequalities in our present system for electing the President. In so doing it will probably surprise those who have always assumed that the present Electoral College system favored the rural citizens from the less populous states because they have a larger number of electoral votes per capita. It also offers quantitative support for serious students of the system who suspected, but could not prove, how the system worked. For example, the American Bar Association has stated that: "[I]t is claimed that the system gives too much weight to some voters and too little to others . . . gives excessive power to organized groups in states where the parties are evenly matched . . . limits campaigns to pivotal states and nominations for the Presidency to persons from large states."<sup>49</sup> The New York Bar Association was of the opinion that:

[T]he present system, among other defects, distorts the entire process because of the undue attention that is given "swing" voters in states with a large number of electors in which the parties are evenly matched. . . . While the ratio of electoral votes to population is such that it would seem that the system favors residents of Alaska, Nevada, Wyoming, and other sparsely populated states the most and New York, Pennsylvania, California and other heavily populated states the least, the practice of giving all of a

48. As previously indicated, see note 14 *supra*, earlier mathematical studies of the Electoral College focused on a different problem. They concerned themselves with the problem of how often the bloc of electoral votes cast by any given state could be decisive or critical in determining the outcome of the presidential election. They found that the voting power (or power index) of each state, considered as an entity, is almost proportional to the number of electoral votes it can cast, with a slight "systematic bias giving an advantage to the larger states." I. MANN & L. SHAPLEY, VALUES OF LARGE GAMES, VI: EVALUATING THE ELECTORAL COLLEGE EXACTLY 8-9 (RAND Corp. Memo. RM-3158-PR 1962). These researchers did not concern themselves at that time with the more difficult problem of the voting power of the individual citizen-voters residing in the different states, although they did speculate about the results of such a study.

It is to be wondered what is the case in the multimillion-person game [problems of this nature are called games], when the voters, rather than the states, are considered the individual players. There is some intuitive evidence that the power indices would again be in favor of the voter in the large states, and that this bias quantitatively might be as much as double the one seen by treating the states as the players [the voting units of the game].

*Id.* at 9.

In preparing this study the author has consulted with these two earlier researchers, each of whom was kind enough to publicly express their support for his very similar analysis of voting power under a multi-member district system. See note 8 *supra*. It should be carefully noted that the author's analysis and the earlier studies are in no way in conflict. This study has used the earlier findings as a basis for determining the voting power of the individual citizens based in part upon the power index of their state. See note 28 *supra*.

49. Commission on Electoral College Reform, American Bar Ass'n, *Electing the President*, 53 A.B.A.J. 219, 220 (1967).

state's electoral votes to the winner of its popular vote, by however small a plurality, has in fact contributed to the parties' selecting their candidates and directing their campaigns with a view toward affecting the outcome in the large industrial states.<sup>50</sup>

The analysis would also seem to support Professor Alexander M. Bickel of Yale Law School who originally thought that no such proof could be possible:

I think it reasonably clear that the effect of the electoral college system over recent generations has been that it wastes many popular votes in relatively homogeneous states, and causes Presidential elections to be decided for the most part in the large, populous, heterogeneous states, where in turn block voting, as by minorities or other interest groups, is often decisive. No one, I concede, can offer mathematical proof that this is how the system has worked and will continue to work, but that is not very important. Precisely, perhaps, because there is no mathematical proof, this — what I have described — is the common perception of the system. Whether or not it may be in some part myth, it governs political behavior. The result has been that modern Presidents have been particularly sensitive and responsive to urban and minority interest. . . .<sup>51</sup>

After extensive hearings on Electoral College reform before the Senate Subcommittee on Constitutional Amendments, a study of the evidence prepared by the staff concluded:

A further charge against the unit-rule system is that it strongly tends to overemphasize the political importance of the large populous states. This has meant that presidential candidates have come almost exclusively from such States. Except for Mr. Landon of Kansas in 1936 and the incumbent President Truman of Missouri in 1948, both major parties have limited their presidential nominations in the last half century to men from the eight largest States. Able men from small States are given little chance to secure nominations from either major party, and are generally not even regarded as "presidential timber." Both major parties are accused of greater concern with the capacity of their candidate to carry certain pivotal States than to command the support of voters throughout the Nation as a whole.

The pivotal State also tends to monopolize the attention of the candidates and their campaign efforts with the result that presidential campaigns are not carried to the Nation as a whole. States which are not regarded as doubtful, or which are considered of less

50. COMMITTEE ON FEDERAL LEGISLATION, BAR ASS'N OF THE CITY OF NEW YORK, PROPOSED CONSTITUTIONAL AMENDMENT PROVIDING FOR DIRECT ELECTION OF PRESIDENT AND VICE PRESIDENT 4 (1967).

51. Statement of Professor Bickel, in *Hearings Before the Senate Subcomm. on Constitutional Amendments*, 90th Cong., 1st Sess. (July 18, 1967).



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importance, are relatively ignored. Citizens in the smaller States are less apt to see or hear the candidates in person and may be inclined to think that their interests are of less importance to the candidates. For the same reason, it is charged that issues, party platforms, and campaign promises are formulated with a view to these pivotal States. . . .

In other words, despite the imbalance in the electoral college favoring small States, the large urban States have come into dominance because of the operation of the unit rule. Most defenders of the present system do not dispute this point.<sup>52</sup>

Even a mathematical demonstration of gross inequalities in voting power under the existing system may not be a sufficient reason to warrant a change. Arguably one may also wish to consider the other interactions between the citizen and his Federal Government, including the Senate which clearly favors the smaller states. Some consideration should also be given to the political aspects of the present system and the effects of a change upon the balance of power among different interest groups. Nevertheless, in a country dedicated to the concepts of democracy and equality for all men, it is only appropriate that a very high priority be placed on assuring every citizen an equal opportunity to participate in the election of a man who will serve as our highest national officer and as a symbol and spokesman equally for all Americans. The effects the existing and proposed systems for electing the President have on the voting rights of all Americans must be very carefully considered by Congress, by the members of the state legislatures, and by the people, who must eventually approve, and ultimately live with, any constitutional provision.<sup>53</sup>

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52. STAFF OF THE SUBCOMMITTEE ON CONSTITUTIONAL AMENDMENTS, COMMITTEE ON THE JUDICIARY, THE ELECTORAL COLLEGE, OPERATION AND EFFECT OF PROPOSED AMENDMENTS TO THE CONSTITUTION OF THE UNITED STATES 31-32 (Comm. Print 1961).

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